Center of Pressure

Objectives:-

1- To determine experimentally the resultant force acting on a partially and fully submerged plane surface.

2- To determine experimentally the moments due to the resultant force ,and compare the results with theoretical analysis.

3- To determine experimental and the theoretical center of pressure.

Description of Apparatus:-

Water is contained in a quadrant of a semi-circular perspex tank assembly which is allowed to roll on a smooth surface. The cylindrical sides of the quadrant have their axes coincident with the centre of rotation of the tank assembly, and therefore the total fluid pressure acting on these surfaces exerts no moment about that centre. The only moment present is that due to the fluid pressure acting on the plane surface. This moment is measured experimentally by applying weights to a weight hanger mounted on the semi-circular assembly on the opposite side to the quadrant tank.

A second tank, Situated on the same side of the assembly as the weight hanger, provides a trimming facility and enables different angles of balance to. The angular position of the plane and the height of water above it are measured on a protractor scale mounted on the tank and a linear scale on the back panel.

Fig. 1 center of pressure apparatus

The apparatus is completed by base leveling feet and spirit level together with a water reservoir and filling jug.

Specifications:

Inner radius R_1 = 100 mm
Outer radius R_2 = 200 mm
Breadth(width) B = 75 mm
Moment arm R_3 = 203 mm
Weight hanger 50 g

Theory:-

Centre of Pressure may be defined as the point in a plane at which the fluid thrust can be said to be acting normal to that plane.

Referring to (fig 2), consider an element at start depth **y**, width d**y**.

The basic equation of the resultant hydrostatic force is as follows:

$$F = \int_A P \times dA \qquad \dots (1)$$

Where,

$$P = \gamma \times (y\cos\theta - h)$$

$$dA = B \times dy$$

Therefore, result force on element :

$$F = \int \gamma \times (y\cos\theta - h) \times B \times dy \qquad \dots(2)$$

Equation (2) provides an expression for the resultant force *F* as a function of the pressure acting over the differential area element *dA = B dy*.

Assume the localized acceleration of gravity is a constant 9.81 m/s^2 and that the water is incompressible with constant density 1000 kg/m^3 so:

$$\gamma = \rho \times g = 1000 \times 9.81 = 9810 \quad N/m^3 \qquad \dots (3)$$

The moment of force on element about **O** is:

$$M_o = F \times y$$

$$M_o = \left\langle \int \gamma \times (y\cos\theta - h) \times B \times dy \right\rangle \times y$$

$$M_o = \left\langle \int \gamma \times (y^2\cos\theta - hy) \times B \times dy \right\rangle \qquad \dots (4)$$

The value of moment experimentally is:

$$M_{exp} = g \times m \times R_3 \qquad \dots (5)$$

$$F_{exp} = m \times g \qquad \dots (6)$$

<u>Case 1:-</u> plane partially submerged

Recall Equ.(2) and integrate from (h secθ) to R_2 :-

$$F_{th} = \gamma\, B \int_{h\sec\theta}^{R_2} (y\cos\theta - h)\, dy$$

$$F_{th} = \gamma\, B \left[\frac{y^2}{2}\cos\theta - hy \right]_{h\sec\theta}^{R_2}$$

$$F_{th} = \gamma\, B \left[\frac{R_2^{\,2}}{2}\cos\theta - hR_2 + \frac{h^2\sec\theta}{2} \right] \qquad \dots(7)$$

Now, recall Equ.(4) and integrate from (h secθ) to R_2 :-

$$M_{th} = \gamma\, B \int_{h\sec\theta}^{R_2} (y^2\cos\theta - hy)\, dy$$

$$M_{th} = \gamma\, B \left[\frac{y^3}{3} \cos\theta - h\, \frac{y^2}{2} \right]_{h \sec\theta}^{R_2}$$

$$M_{th} = \gamma\, B \left[\frac{R_2^{\,3} \cos\theta}{3} - \frac{R_2^{\,2}\, h}{2} + \frac{h^3 \sec^2\theta}{6} \right] \qquad \ldots(8)$$

$$M_{th} + \frac{\gamma\, B\, R_2^{\,2}\, h}{2} = \gamma\, B \left[\frac{R_2^{\,3} \cos\theta}{3} + \frac{h^3 \sec^2\theta}{6} \right] \qquad \ldots(9)$$

Case 2:- ***plane fully submerged***

Recall Equ.(2) and integrated from R_1 to R_2

$$F_{th} = \gamma\, B \int_{R_1}^{R_2} (y \cos\theta - h)\, dy$$

$$F_{th} = \gamma\, B \left[\frac{y^2}{2} \cos\theta - hy \right]_{R_1}^{R_2}$$

$$F_{th} = \gamma\, B \left[\frac{\cos\theta}{2} \left(R_2^{\,2} - R_1^{\,2}\right) - h\,(R_2 - R_1) \right] \qquad \ldots(10)$$

Now, recall Equ.(4) and integrated from R_1 to R_2

$$M_{th} = \gamma\, B \int_{R_1}^{R_2} \left(y^2 \cos\theta - hy\right) dy$$

$$M_{th} = \gamma\, B \left[\frac{y^3}{3} \cos\theta - h\, \frac{y^2}{2} \right]_{R_1}^{R_2}$$

$$M_{th} = \gamma\, B \left[\frac{\cos\theta}{3} \left(R_2^{\,3} - R_1^{\,3}\right) - \frac{h}{2} \left(R_2^{\,2} - R_1^{\,2}\right) \right] \qquad \ldots(11)$$

Center of pressure:-

$$\left(Y_{c.p}\right)_{th} = \frac{M_{th}}{F_{th}} \qquad \ldots(12)$$

$$\left(Y_{c.p}\right)_{exp} = \frac{M_{exp}}{F_{exp}} \qquad \ldots(13)$$

$$\textit{Percentage of error} = \left| \frac{\left(Y_{c.p}\right)_{th} - \left(Y_{c.p}\right)_{exp}}{\left(Y_{c.p}\right)_{th}} \right| \times 100\,\% \qquad \ldots(14)$$

Procedure:-

1-Affix the weight hanger to the cord. The apparatus will now require trimming in order to bring the submerged plane to the vertical (i.e. 0^0 position). This is achieved by gently pouring water into the

trim tank until the desired position is achieved. The protractor on the tank assembly should be read against the zero line on the back scale.

2- First add a 20g weight to the weight hanger, Pour water into the quadrant tank until a 0^0 balance is restored. Note the weight and the height reading of the water (h). Repeat the procedure for the fuIl range of weights in steps of 50 g.

3-Empty both tanks of water and repeat the procedure for 10^0 , 20^0.

4-Readings should be tabulated in table (1) and(2).

Discussion:

1-The point of action of the result force is below the centroid, why?

2-If we change the water to some other fluid ,such as oil , does the magnitude of the result force and its location change? why?

3-What is the applications of this experiment?

4-Explane why the tank take the shape of quadrant?

5-Discuss the sources of error?

6-Plot ,find the slope, and discuss the results for both cases :

-partially submerged

$[M_{th} + \frac{\gamma B R_2^2 h}{2}$ against $h^3]$ and $[M_{exp} + \frac{\gamma B R_2^2 h}{2}$ against $h^3]$

-fully submerged

$[M_{th}$ against h] and $[M_{exp}$ against h]

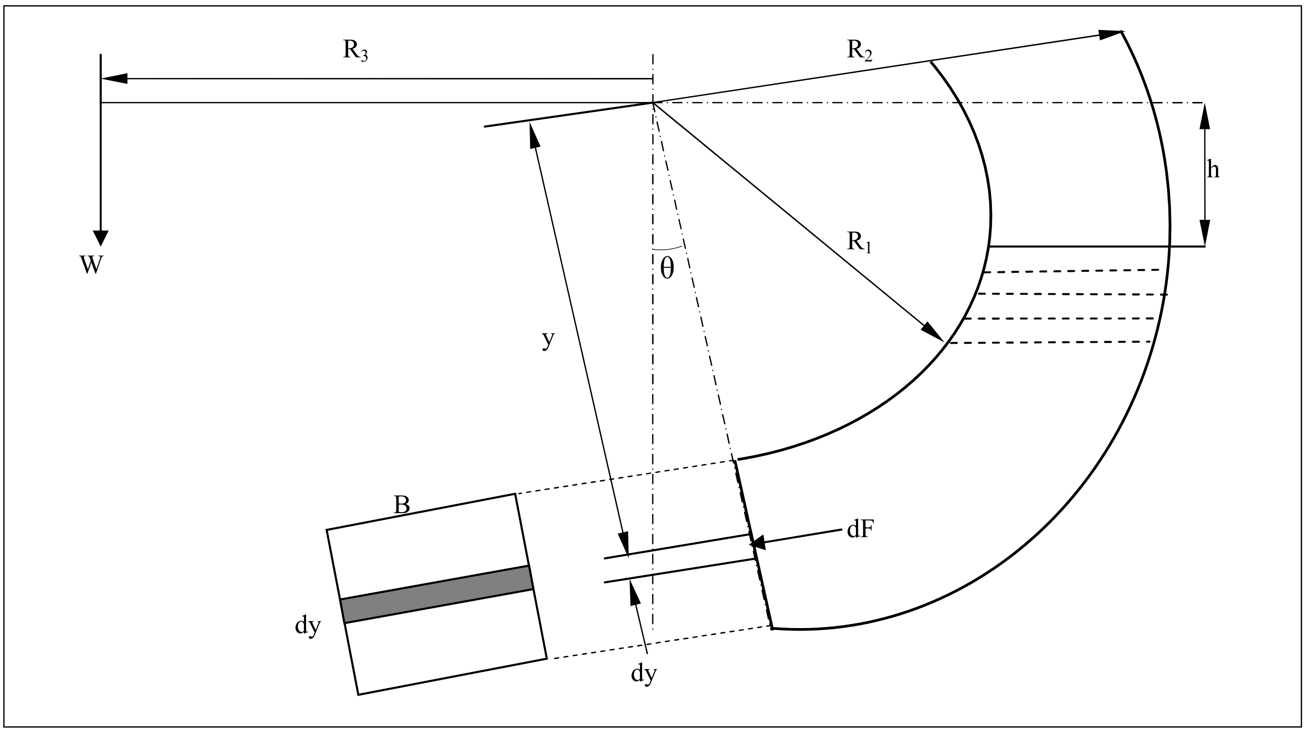

Fig. 2 schema of center of pressure apparatus

Table (1):- partially submerged

m (g)	h (mm)	h^3 (m^3)	F_{th} (N)	M_{th} (N.m)	$M_{th} + \frac{\gamma B R_2^2 h}{2}$	F_{exp} (N)	M_{exp} (N.m)	$M_{exp} + \frac{\gamma B R_2^2 h}{2}$	$(Y_{c.p})_{th}$ (m)	$(Y_{c.p})_{exp}$ (m)	Error %

Table(2):- fully submerged

m (g)	h (mm)	h (m)	F_{th} (N)	M_{th} (N.m)	F_{exp} (N)	M_{exp} (N.m)	$(Y_{c.p})_{th}$ (m)	$(Y_{c.p})_{exp}$ (m)	Error %